

Genuine Multiparty Quantum Entanglement Suppresses Multiport Classical Information Transmission

R. Prabhu, Aditi Sen(De), and Ujjwal Sen

Harish-Chandra Research Institute, Chhatnag Road, Jhansi, Allahabad 211 019, India

We establish a universal complementarity relation between the capacity of classical information transmission by employing a multiparty quantum state as a multiport quantum channel, and the genuine multipartite entanglement of the quantum state. The classical information transfer is from a sender to several receivers by using the quantum dense coding protocol with the multiparty quantum state shared between the sender and the receivers. The relation holds for arbitrary pure or mixed quantum states of an arbitrary number of parties in arbitrary dimensions.

I. INTRODUCTION

The discoveries of the quantum communication strategies and further work in that direction over last two decades, beginning with the protocols of quantum dense coding [1], quantum teleportation [2] and quantum key distribution [3], have revolutionized the way we think about modern communication schemes. The importance of such protocols lies in the fact that they can efficiently transmit classical or quantum information in a way that is better than what is possible by using classical protocols. Such quantum communication schemes have already been realized experimentally, and in particular, experimental quantum dense coding have been reported in several systems including photonic states, ion traps, and nuclear magnetic resonance [4]. The protocols were initially introduced for communication between two separated parties. This is true for the theoretical discussions of these protocols as well as for the experimental demonstrations of the same. It has already been established that in the case of such bipartite communication schemes between a sender and a single receiver, shared quantum correlations play a key role.

Commercialization of these protocols demand the implementations of these protocols in a multipartite scenario [5]. Classical information transmission, in the form of quantum dense coding, has already been introduced in multiparticle systems [6]. Further work in this direction include Refs. [7].

In this paper, we consider the multiport quantum channel for transmitting classical information where a sender wishes to send classical information to several receivers by employing the quantum dense coding protocol with a multiparty shared quantum state between all the partners. We find that the quantum advantage in this quantum dense coding scheme is suppressed by the genuine multipartite entanglement of the shared multiparty quantum state. The capacity of multiport quantum dense coding has a complementarity with the amount of shared genuine multisite entanglement. The complementarity is first demonstrated by using the genuine multisite relative entropy of entanglement [8], and holds for arbitrary (pure or mixed) quantum states of an arbitrary number of parties and in arbitrary dimensions. To show that the complementarity is potentially generic, as we go on

to demonstrate the complementarity for an independent measure of genuine multisite entanglement, the generalized geometric measure [9], in which case the relation holds for pure multisite quantum states of an arbitrary number of parties in arbitrary dimensions. We also show that the latter relation holds for mixed states of three qubits.

The rest of the paper is presented as follows. Sections II and III present the definitions required in the paper. In Sec. II, we formally describe the dense coding capacity and the corresponding quantum advantage over its classical counterpart. In Sec. III, we provide brief definitions of the genuine multisite entanglements used. The results are presented in Sec. IV. We provide a conclusion in Sec. V.

II. QUANTUM DENSE CODING CAPACITY AND THE QUANTUM ADVANTAGE

Quantum dense coding (DC) is a quantum communication protocol by which one can transmit classical information encoded in a quantum system from a sender to a receiver [1]. The available resources for the transmission are a shared quantum state and a noiseless quantum channel to transmit the sender's part of the shared quantum state to the receiver's end. If the sender, called Alice, and the receiver, called Bob, share a bipartite quantum state ϱ_{AB} , then the amount of classical information (in bits) that the sender can send to the receiver is given by [6, 10, 11]

$$\mathcal{C}(\varrho_{AB}) = \max\{\log_2 d_A, \log_2 d_A + S(\varrho_B) - S(\varrho_{AB})\}, \quad (1)$$

where d_A is the dimension of Alice's Hilbert space and ϱ_B is the local density matrix of Bob's subsystem. $S(\cdot)$ denotes the von Neumann entropy of its argument and is defined as $S(\sigma) = -\text{tr}(\sigma \log_2 \sigma)$, for an arbitrary quantum state σ . The capacity $\mathcal{C}(\varrho_{AB})$ reaches its maximum when Alice and Bob share a maximally entangled state, which is a pure state with completely mixed local density matrices. Without using the shared quantum state but using the noiseless quantum channel, Alice will be able to send $\log_2 d_A$ bits. This process of sending classical information without using the shared quantum state is referred to as the "classical protocol". Using the shared quantum

state is therefore advantageous if $S(\varrho_B) - S(\varrho_{AB}) > 0$. A bipartite quantum state is said to be dense-codeable in that case. Correspondingly, the quantity

$$\mathcal{C}_{adv}(\varrho_{AB}) = \max\{S(\varrho_B) - S(\varrho_{AB}), 0\} \quad (2)$$

is identified as the “quantum advantage” in a dense coding protocol from Alice to Bob. Note that

$$\mathcal{C}(\varrho_{AB}) = \log_2 d_A + \mathcal{C}_{adv}(\varrho_{AB}). \quad (3)$$

Let us now move on to a multipoint situation and suppose now that there are $N + 1$ parties who share a quantum state $\varrho_{AB_1 B_2 \dots B_N}$. Moreover, we assume that among the $N + 1$ parties, A is the sender and others, i.e. B_1, B_2, \dots, B_N are the receivers. Let us consider a situation where A wants to send, individually, classical information to the B_i ’s ($i = 1, 2, \dots, N$). In this multipoint case, the quantum advantage is naturally defined as

$$\mathcal{C}_{adv}^{max}(\varrho_{AB_1 B_2 \dots B_N}) = \max\{\mathcal{C}_{adv}(\varrho_{AB_i}) | i = 1, 2, \dots, N\}, \quad (4)$$

where ϱ_{AB_i} is the local density matrix of A and B_i ($i = 1, 2, \dots, N$). This “multipoint dense coding quantum advantage” quantifies the amount of classical information that can be sent from the sender A to the N receivers by using the quantum dense coding protocol with the shared quantum state $\varrho_{AB_1 B_2 \dots B_N}$, over and above the amount of classical information that can be sent by using a classical protocol.

III. GENUINE MULTIPARTITE ENTANGLEMENT MEASURES

In this section, we will introduce two multipartite entanglement measures which can quantify genuine multipartite entanglement of an arbitrary quantum state $\varrho_{A_1 A_2 \dots A_n}$ shared between n parties.

A. Genuine Multisite Relative Entropy of Entanglement

The relative entropy of entanglement was first proposed as a measure of entanglement for an arbitrary bipartite state ϱ_{AB} [8], and is given by

$$E_R(\varrho_{AB}) = \min_{\sigma \in \text{sep}} S(\varrho || \sigma). \quad (5)$$

Here, “sep” is the set of all separable states in $A : B$, and the relative entropy, $S(\varrho || \sigma)$, between ϱ and σ , is defined as $S(\varrho || \sigma) = \text{tr}(\varrho \log_2 \varrho - \varrho \log_2 \sigma)$. It was shown that the relative entropy of entanglement satisfies all the properties which is required of a “good” entanglement measure [8]. E_R reaches its maximum for a maximally entangled state.

In a multipartite scenario, to quantify genuine multipartite entanglement of arbitrary multipartite quantum

(pure or mixed) states, the genuine multipartite relative entropy of entanglement can be defined as

$$E_R^G(\varrho_{A_1 A_2 \dots A_n}) = \min_{\sigma \in \text{n-gen}} S(\varrho_{A_1 A_2 \dots A_n} || \sigma), \quad (6)$$

where “n-gen” is the set of all n -party multipartite quantum (pure or mixed) states which are not genuinely multipartite entangled. A multipartite quantum state is said to be genuinely multipartite entangled if it cannot be written as a probabilistic mixture of multipartite quantum states which are separable across at least one bipartition of the n parties. As an example, for three-party quantum systems between A_1 , A_2 , and A_3 , a probabilistic mixture of two quantum states which are respectively separable across $A_1 : A_2 A_3$ and $A_2 : A_1 A_3$ is not genuinely multisite entangled. E_R^G is the “relative entropy distance” of the corresponding multisite state from the convex set of all multipartite states which are not genuinely multipartite entangled.

B. Generalized Geometric Measure

The generalized geometric measure [9] was first proposed to be a measure of genuine multipartite entanglement of a pure multipartite quantum state, by using a distance function of the given pure state from all pure multisite states which are not genuinely multipartite entangled, and is defined as

$$\mathcal{E}(|\psi\rangle_{A_1 A_2 \dots A_n}) = \min(1 - |\langle \phi | \psi \rangle|^2), \quad (7)$$

where the minimization is over all $|\phi\rangle_{A_1 A_2 \dots A_n}$ that are not genuinely multipartite entangled.

This definition can be extended to arbitrary multipartite mixed quantum states by using the convex roof approach. Therefore, the generalized geometric measure for an arbitrary mixed quantum state can be defined as

$$\mathcal{E}(\varrho_{A_1 A_2 \dots A_n}) = \min_i \sum_i p_i \mathcal{E}(|\psi_i\rangle_{A_1 A_2 \dots A_n}), \quad (8)$$

where the minimization is performed over all pure state decompositions of $\varrho_{A_1 A_2 \dots A_n} = \sum_i p_i (|\psi_i\rangle\langle\psi_i|)_{A_1 A_2 \dots A_n}$.

IV. COMPLEMENTARITY

In this section, we establish a complementarity relation between the amount of classical information that can be sent through the multisite quantum state $\varrho_{AB_1 B_2 \dots B_N}$, as quantified by the multipoint dense coding quantum advantage (\mathcal{C}_{adv}^{max}), as defined in Sec. II, with genuine multipartite entanglement measures – genuine multipartite relative entropy of entanglement and generalized geometric measure.

A. Multiport dense coding advantage vs genuine multiparty relative entropy of entanglement

Let A, B_1, B_2, \dots, B_N be $N + 1$ observers who share the state $\varrho_{AB_1B_2\dots B_N}$, which is an arbitrary $(N + 1)$ party (pure or mixed) quantum state of arbitrary dimensions. In the multiport dense coding protocol that we consider, we assume that A is the sender and the B_i 's ($i = 1, 2, \dots, N$) are the receivers. We now prove the following complementarity between the quantum advantage in this multiport scenario with the genuine multiparty relative entropy of entanglement.

Theorem 1: *For the arbitrary multipartite pure or mixed quantum state $\varrho_{AB_1B_2\dots B_N}$ in arbitrary dimensions, the genuine multiparty relative entropy of entanglement and the multiport dense coding quantum advantage satisfy*

$$\mathcal{C}_{adv}^{max} + E_R^G \leq \log_2 d, \quad (9)$$

where d is the maximal dimension of the Hilbert spaces of the B_i 's.

Proof. We have

$$\begin{aligned} E_R^G(\varrho_{AB_1B_2\dots B_N}) &= \min_{\sigma \in \text{sep}} S(\varrho||\sigma) \\ &\leq \min_{\sigma \in \text{sep}_1} S(\varrho||\sigma') \\ &\equiv E_R^{AB_1:\text{rest}}(\varrho_{AB_1:B_2\dots B_N}) \\ &\leq E_f^{AB_1:\text{rest}}(\varrho_{AB_1:B_2\dots B_N}) \\ &\leq S(\varrho_{AB_1}). \end{aligned} \quad (10)$$

Here the set “sep₁” is the set of quantum states of A, B_1, B_2, \dots, B_N which are separable across $AB_1 : B_2 \dots B_N$ bipartition. $E_R^{AB_1:\text{rest}}(\varrho_{AB_1:B_2\dots B_N})$ and $E_f^{AB_1:\text{rest}}(\varrho_{AB_1:B_2\dots B_N})$ respectively denote the relative entropy of entanglement and the entanglement of formation of the state $\varrho_{AB_1B_2\dots B_N}$ in the same bipartition. The second inequality is obtained due to the fact that the relative entropy of entanglement is bounded above by the entanglement of formation [12], while the third follows from the fact that the von Neumann entropy of the local density matrix is an upper bound of the entanglement of formation for bipartite states [13].

For the multipartite state $\varrho_{AB_1B_2\dots B_N}$, the dense coding advantage can be written as

$$\mathcal{C}_{adv}^{max}(\varrho_{AB_1B_2\dots B_N}) = \max\{S_{B_1} - S_{AB_1}, S_{B_2} - S_{AB_2}, \dots, S_{B_N} - S_{AB_N}, 0\}. \quad (11)$$

where $S_{B_i} = S(\varrho_{B_i})$ and $S_{AB_i} = S(\varrho_{AB_i})$ are the single-site and two-site von Neumann entropies of $\varrho_{AB_1B_2\dots B_N}$ respectively. Consider the instance when $S_{B_1} - S_{AB_1}$ attains the maximum. Then

$$\mathcal{C}_{adv}^{max}(\varrho_{AB_1B_2\dots B_N}) = S_{B_1} - S_{AB_1}. \quad (12)$$

Adding Eqns. (10) and (12), we obtain

$$\begin{aligned} \mathcal{C}_{adv}^{max}(\varrho_{AB_1B_2\dots B_N}) + E_R^G(\varrho_{AB_1B_2\dots B_N}) &\leq S_{B_1} \\ &\leq \log_2 d_{B_1}. \end{aligned} \quad (13)$$

If the maximum is obtained for the pair, say AB_i , in \mathcal{C}_{adv}^{max} , we have to choose the corresponding bipartition in the relative entropy of entanglement in Eq. (10), and in that case, the complementarity relation will be bounded above by the logarithm of the dimension of that B_i . Therefore, we finally have $\max\{\log_2 d_{B_1}, \log_2 d_{B_2}, \dots, \log_2 d_{B_N}\}$, which is denoted as $\log_2 d$, as the upper bound for the sum $\mathcal{C}_{adv}^{max} + E_R^G$. Hence the proof. ■

The complementarity relation which is established above clearly indicates that a high genuine multipartite entanglement will lower the advantage of the same state for transmitting classical information.

B. Relation between dense coding advantage and GGM

Towards showing that the obtained complementarity relation is generic, we consider another genuine multiparty entanglement measure, the generalized geometric measure, defined in Sec. III. The relation is first proven for pure multiparty quantum states in arbitrary dimensions. For simplicity, we assume that the state lies in $(\mathbb{C}^d)^{\otimes N+1}$, with arbitrary dimension d . Subsequently we show that the relation also holds for arbitrary mixed states, $\rho_{AB_1B_2\dots B_N}$, whose two-party reduced density matrices are of rank 3 or less.

Consider therefore the $(N + 1)$ -party pure state, $|\psi\rangle_{AB_1B_2\dots B_N}$, which is employed by the sender A to perform dense coding with the receivers B_i 's ($i = 1, 2, \dots, N$).

Theorem 2: *The sum of the advantage in dense coding and the generalized geometric measure for the arbitrary pure state $|\psi\rangle_{AB_1B_2\dots B_N}$ is bounded above by unity, i.e.,*

$$\frac{1}{\log_2 d} \mathcal{C}_{adv}^{max} + \frac{d}{d-1} \mathcal{E} \leq 1. \quad (14)$$

Remark. Note that the factors $\frac{1}{\log_2 d}$ and $\frac{d}{d-1}$, respectively for dense coding advantage and GGM, are normalizations that make the individual terms have maximal values as unity.

Proof. Let us assume, without loss of generality, that the maximum in the multiport quantum advantage in dense coding is attained for $S_{B_1} - S_{AB_1}$.

$$\mathcal{C}_{adv}^{max}(|\psi\rangle_{AB_1B_2\dots B_N}) = S_{B_1} - S_{AB_1} \quad (15)$$

We now note that the GGM for the state $|\psi\rangle_{AB_1B_2\dots B_N}$ can be shown to be given by [9]

$$\mathcal{E}(|\psi\rangle_{AB_1B_2\dots B_N}) = 1 - \max\{\Lambda_j\}, \quad (16)$$

where Λ_j 's are the maximum eigenvalues of the local density matrices of all possible bipartite partitions of the state $|\psi\rangle_{AB_1B_2\dots B_N}$. Therefore, we have

$$\mathcal{E}(|\psi\rangle_{AB_1B_2\dots B_N}) \leq 1 - \lambda_{AB_1}, \quad (17)$$

with λ_{AB_1} being the maximum eigenvalue of the two-party reduced density matrix ϱ_{AB_1} of $|\psi\rangle_{AB_1B_2\dots B_N}$. From Eqs. (15) and (17), we obtain

$$\begin{aligned} \frac{\mathcal{C}_{adv}^{max}(|\psi\rangle_{AB_1B_2\dots B_N})}{\log_2 d} + \frac{d}{d-1}\mathcal{E}(|\psi\rangle_{AB_1B_2\dots B_N}) \\ \leq \frac{S_{B_1}}{\log_2 d} - \frac{S_{AB_1}}{\log_2 d} + \frac{d(1 - \lambda_{AB_1})}{d-1}. \end{aligned}$$

The von Neumann entropy of ϱ_{AB_1} is related to its highest eigenvalue as $S_{AB_1} \geq \log_2(\frac{1}{\lambda_{AB_1}})$ [14]. Using this inequality, the relation reduces to

$$\begin{aligned} \frac{\mathcal{C}_{adv}^{max}(|\psi\rangle_{AB_1B_2\dots B_N})}{\log_2 d} + \frac{d}{d-1}\mathcal{E}(|\psi\rangle_{AB_1B_2\dots B_N}) \\ \leq \frac{S_{B_1}}{\log_2 d} + \frac{\log_2 \lambda_{AB_1}}{\log_2 d} + \frac{d(1 - \lambda_{AB_1})}{d-1}. \end{aligned}$$

It can be shown that the sum of the second and third terms in the right-hand-side of the above inequality is always negative, for $d > 2$ and it vanishes for $d = 2$. The proof follows from the fact that $S_{B_1} \leq \log_2 d$. ■

We now consider quantum states of $N + 1$ parties which are possibly mixed, and are in arbitrary dimensions. For simplicity, we assume that the state is defined on $(\mathbb{C}^d)^{\otimes N+1}$, with arbitrary dimension d .

Theorem 3: *For the arbitrary (possibly mixed) quantum state $\varrho_{AB_1B_2\dots B_N}$, whose local density matrices ϱ_{AB_i} are of rank three or lower, the sum of the normalized quantum advantage in dense coding and the normalized generalized geometric measure is bounded above by unity.*

Proof. The genuine multiparty entanglement measure, GGM, of $\varrho_{AB_1B_2\dots B_N}$ is given by

$$\mathcal{E}(\varrho_{AB_1B_2\dots B_N}) = \sum_k p_k \mathcal{E}(|\psi_k\rangle\langle\psi_k|), \quad (18)$$

where the ensemble $\{p_k, |\psi_k\rangle\}$ forms the optimal pure state decomposition of the state $\varrho_{AB_1B_2\dots B_N}$ for obtaining the minimum in the convex roof of the GGM. Suppose that $\lambda_{AB_1}^k$ is the maximum eigenvalue of the two-party reduced density matrix, of the parties A and B_1 , of the state $|\psi_k\rangle$. Then we have

$$\mathcal{E}(\varrho_{AB_1B_2\dots B_N}) \leq \sum_k p_k (1 - \lambda_{AB_1}^k).$$

For the cases when the states ϱ_{AB_i} are of at most rank three, one can show that

$$\begin{aligned} \frac{d}{d-1}\mathcal{E}(\varrho_{AB_1B_2\dots B_N}) &\leq \sum_k p_k \frac{S(\text{tr}_{B_2\dots B_N} |\psi_k\rangle\langle\psi_k|)}{\log_2 d} \\ &\leq \frac{S(\varrho_{AB_1})}{\log_2 d}. \end{aligned} \quad (19)$$

We have used concavity of the von-Neumann entropy to get the last inequality. Now, along with the quantum

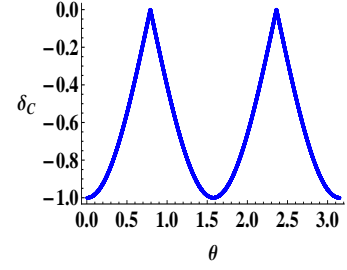


FIG. 1. (Color online.) The complementarity quantity, $\delta_C = \mathcal{C}_{adv}^{max} + \mathcal{E} - 1$ is plotted against the state parameter, θ , of the generalized GHZ state. The horizontal axis is in radians, while the vertical axis is dimensionless.

advantage in dense coding defined in the Eq. (4), and using the above inequality, we obtain

$$\begin{aligned} \frac{\mathcal{C}_{adv}^{max}(\varrho_{AB_1B_2\dots B_N})}{\log_2 d} + \frac{d}{d-1}\mathcal{E}(\varrho_{AB_1B_2\dots B_N}) \\ \leq \frac{S_{B_1} - S_{AB_1}}{\log_2 d} + \frac{S_{AB_1}}{\log_2 d} \leq 1. \end{aligned} \quad (20)$$

Here we have assumed, without loss of generality, that the maximum in the definition of \mathcal{C}_{adv}^{max} is attained by the AB_1 pair. ■

We now illustrate that the obtained complementary relation between the multiport dense coding quantum advantage and genuine multipartite entanglement is tight. For this investigation, we consider two important classes of tripartite pure quantum states of three qubits – the generalized Greenberger-Horne-Zeilinger (GHZ) states [15] and the generalized W states [16].

Let us first consider the generalized GHZ state [15] given by

$$|\psi_{GHZ}\rangle = \cos \theta |000\rangle + \sin \theta |111\rangle, \quad (21)$$

where $\theta \in (0, \pi)$. We plot the complementarity quantity

$$\delta_C = \frac{1}{\log_2 d} \mathcal{C}_{adv}^{max} + \frac{d}{d-1} \mathcal{E} - 1 \quad (22)$$

with respect to the state parameter θ (see Fig. 1). We see that the saturation of the complementarity relation occurs at $\theta = \frac{\pi}{4}$ and $\frac{3\pi}{4}$. Note here that the complementarity relation of Theorem 2 implies that δ_C is always non-positive, and that the vanishing of δ_C implies that the bound is tight.

Similarly, we consider the generalized W states, which are known to be inequivalent to the generalized GHZ states by stochastic local operations and classical communication [16], given by

$$|\psi_W\rangle = \sin \theta' \cos \phi' |011\rangle + \sin \theta' \sin \phi' |101\rangle + \cos \theta' |110\rangle, \quad (23)$$

with $\theta' \in (0, \pi)$ and $\phi' \in (0, 2\pi)$. In Fig. 2, we depict the projection of δ_C on the (θ', ϕ') -plane for the generalized W state. It clearly indicates that there are regions where the complementarity relation obtained in Theorem 2 is saturated.

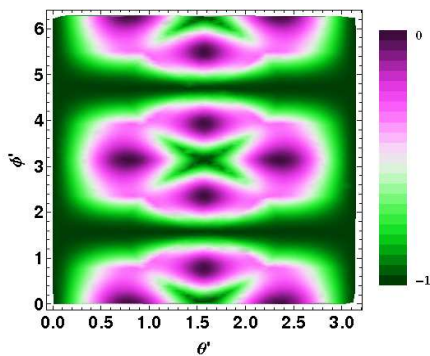


FIG. 2. (Color online.) Saturation of the complementarity quantity. The figure presents the projection of the δ_C surface, for the generalized W state, on the plane of the state parameters θ' and ϕ' . The dark violet regions show the area where δ_C vanishes, and hence the complementarity relation is saturated. The axes are in radians.

V. CONCLUSION

Summarizing, we have established a complementary relationship between the quantum advantage of the multiport classical capacity of a multipartite quantum state used as a quantum channel and the genuine multipartite entanglement of the multipartite state. The relation is demonstrated for two genuine multipartite entanglement measures – the genuine multipartite relative entropy

of entanglement and the generalized geometric measure. The relation holds for pure or mixed quantum states of arbitrary dimensions and of an arbitrary number of parties.

Quantum dense coding in the case of a sender and single receiver is known to provide the maximal quantum advantage for the maximally entangled bipartite quantum state. Due to the monogamy of quantum correlations, one intuitively expects that a high multipartite entanglement of a quantum state will suppress the reduced bipartite entanglements, and hence will reduce the capacity of multiport dense coding. However, since a quantitative statement of the constraint on the multipartite entanglement due to the monogamy of bipartite entanglement is as yet missing, it is not straightforward to relate the monogamy of bipartite quantum correlations with a multiport quantum advantage of channel capacities. The complementarity relation derived in this paper can shed light towards a quantitative understanding in this direction.

ACKNOWLEDGMENTS

R.P. acknowledges support from the Department of Science and Technology, Government of India, in the form of an INSPIRE faculty scheme at the Harish-Chandra Research Institute, India.

-
- [1] C.H. Bennett and S.J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
 - [2] C.H. Bennett, G. Brassard, C. Crépeau, R. Josza, A. Peres, and W.K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
 - [3] C.H. Bennett and G. Brassard, in *Proceedings of the International Conference on Computers, Systems and Signal Processing, Bangalore, India* (IEEE, NY, 1984).
 - [4] K. Mattle, H. Weinfurter, P.G. Kwiat, and A. Zeilinger, Phys. Rev. Lett. **76**, 4656 (1996); X. Fang, X. Zhu, M. Feng, X. Mao, and F. Du, Phys. Rev. A **61**, 022307 (2000); X. Li, Q. Pan, J. Jing, J. Zhang, C. Xie, and K. Peng, Phys. Rev. Lett. **88**, 047904 (2002); J. Jing, J. Zhang, Y. Yan, F. Zhao, C. Xie, and K. Peng, Phys. Rev. Lett. **90**, 167903 (2003); T. Schaetz, M.D. Barrett, D. Leibfried, J. Chiaverini, J. Britton, W.M. Itano, J.D. Jost, C. Langer, and D.J. Wineland, Phys. Rev. Lett. **93**, 040505 (2004); J.T. Barreiro, T.-C. Wei, and P.G. Kwiat, Nature Physics **4**, 282 (2008);
 - [5] For a recent review, see e.g. A. Sen(De) and U. Sen, Physics News **40**, 17 (2010) (arXiv:1105.2412).
 - [6] D. Bruß, G.M. D'Ariano, M. Lewenstein, C. Macchiavello, A. Sen(De), and U. Sen, Phys. Rev. Lett. **93**, 210501 (2004); D. Bruß, M. Lewenstein, A. Sen(De), U. Sen, G.M. D'Ariano, and C. Macchiavello, Int. J. Quant. Inf. **4**, 415 (2006).
 - [7] M. Horodecki and M. Piani, J. Phys. A: Math. Theor. **45**, 105306 (2012); G. Wang and M. Ying, Phys. Rev. A **77**, 032306 (2008); Z. Shadman, H. Kampermann, C. Macchiavello, and D. Bruß, Phys. Rev. A **85**, 052306 (2012); R. Prabhu, A.K. Pati, A. Sen(De), and U. Sen, arXiv:1203.4114, and references therein.
 - [8] V. Vedral, M.B. Plenio, M.A. Rippin, and P.L. Knight, Phys. Rev. Lett. **78**, 2275 (1997); V. Vedral, Rev. Mod. Phys. **74**, 197 (2002).
 - [9] A. Sen(De) and U. Sen, Phys. Rev. A **81**, 012308 (2010); A. Sen(De) and U. Sen, arXiv:1002.1253 [quant-ph].
 - [10] J.P. Gordon, in *Proceedings of the International School of Physics Enrico Fermi, Course XXXI*, edited by P. A. Miles (Academic Press, New York, 1964), p. 156; L.B. Levitin, in *Proceedings of the VI National Conference Information Theory*, Tashkent, 1969, p. 111; A.S. Holevo, Prob. Peredachi Inf. **9**, 3 (1973) [Probl. Infor. Transm. **9**, 110 (1973)]; B. Schumacher and M.D. Westmoreland, Phys. Rev. A **56**, 131 (1997); A.S. Holevo, IEEE Trans. Inf. Theory **44**, 269 (1998).
 - [11] S. Bose, M.B. Plenio, and V. Vedral, J. Mod. Optics. **47**, 291 (2000); T. Hiroshima, J. Phys. A: Math. Gen. **34**, 6907 (2001); G. Bowen, Phys. Rev. A **63**, 022302 (2001); M. Horodecki, P. Horodecki, R. Horodecki, D. Leung, and B. Terhal, Quantum Information and Computation **1**, 70 (2001); M. Ziman and V. Bužek, Phys. Rev. A **67**, 042321 (2003).
 - [12] V. Vedral and M.B. Plenio, Phys. Rev. A **57**, 1619 (1998).
 - [13] C.H. Bennett, D.P. DiVincenzo, J.A. Smolin, and W.K. Wootters, Phys. Rev. A **54**, 3824 (1996).

- [14] A. Wehrl, Rev. Mod. Phys. **50** 221 (1978).
- [15] D.M. Greenberger, M.A. Horne, and A. Zeilinger, in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, ed. M. Kafatos (Kluwer Academic, Dordrecht, 1989).
- [16] A. Zeilinger, M.A. Horne, and D.M. Greenberger, in *Proc. Squeezed States & Quantum Uncertainty*, eds. D. Han, Y.S. Kim, and W.W. Zachary, NASA Conf. Publ. 3135 (1992); W. Dür, G. Vidal, and J.I. Cirac, Phys. Rev. A **62**, 062314 (2000).